AQA
Realising potential

2021 ASSESSMENT MATERIALS

A- level
MATHS

Sequences and series (Topic D)

Total number of marks: 41

1 The first three terms, in ascending powers of x, of the binomial expansion of $(9+2x)^{\frac{1}{2}}$ are given by

$$(9+2x)^{\frac{1}{2}} \approx a + \frac{x}{3} - \frac{x^2}{54}$$

where a is a constant.

1 (a) State the range of values of x for which this expansion is valid.

Circle your answer.

[1 mark]

$|x| < \frac{2}{9}$ $\qquad$ $|x| < \frac{2}{3}$ $\qquad$ $|x| < 1$ $\qquad$ $|x| < \frac{9}{2}$

1 (b) Find the value of a.

Circle your answer.

[1 mark]

1 $\qquad$ 2 $\qquad$ 3 $\qquad$ 9

7 Consecutive terms of a sequence are related by

$$u_{n+1} = 3 - (u_n)^2$$

7 (a) In the case that $u_1 = 2$

7 (a) (i) Find u_3

[2 marks]

7 (a) (ii) Find u_{50}

[1 mark]

7 (b) State a different value for u_1 which gives the same value for u_{50} as found in part **(a)(ii)**.

[1 mark]

9 An arithmetic sequence has first term a and common difference d.

The sum of the first 36 terms of the sequence is equal to the square of the sum of the first 6 terms.

9 (a) Show that $4a + 70d = 4a^2 + 20ad + 25d^2$

[4 marks]

9 (b) Given that the sixth term of the sequence is 25, find the smallest possible value of a.

[5 marks]

8 $\mathrm{P}(n) = \sum_{k=0}^{n} k^3 - \sum_{k=0}^{n-1} k^3$ where n is a positive integer.

8 (a) Find $\mathrm{P}(3)$ and $\mathrm{P}(10)$

[2 marks]

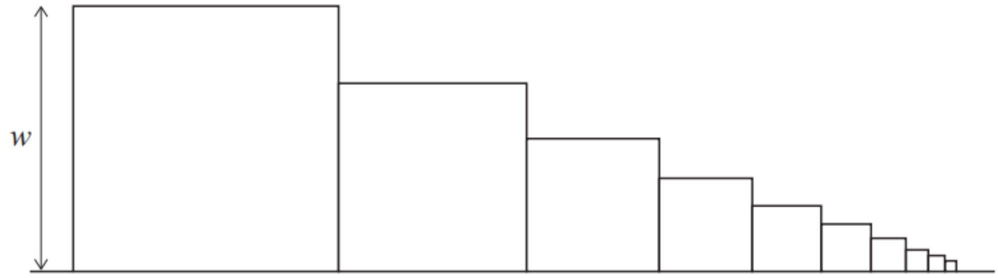
8 (b) Solve the equation $\mathrm{P}(n) = 1.25 \times 10^8$

[2 marks]

3 Find the coefficient of x^2 in the binomial expansion of $\left(2x - \frac{3}{x}\right)^8$

[3 marks]

9 Helen is creating a mosaic pattern by placing square tiles next to each other along a straight line.



The area of each tile is half the area of the previous tile, and the sides of the largest tile have length w centimetres.

9 (a) Find, in terms of w, the length of the sides of the second largest tile.

[1 mark]

9 (b) Assume the tiles are in contact with adjacent tiles, but do not overlap.

Show that, no matter how many tiles are in the pattern, the total length of the series of tiles will be less than $3.5w$.

[4 marks]

9 (c) Helen decides the pattern will look better if she leaves a 3 millimetre gap between adjacent tiles.

Explain how you could refine the model used in part **(b)** to account for the 3 millimetre gap, and state how the total length of the series of tiles will be affected.

[2 marks]

6 (a) Find the first three terms, in ascending powers of x, of the binomial expansion of $\dfrac{1}{\sqrt{4+x}}$

[3 marks]

6 (b) Hence, find the first three terms of the binomial expansion of $\dfrac{1}{\sqrt{4-x^3}}$

[2 marks]

7 (a) Using ${}^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$ show that ${}^{n}C_{2} = \dfrac{n(n-1)}{2}$

[2 marks]

7 (b) (i) Show that the equation

$$2 \times {}^{n}C_{4} = 51 \times {}^{n}C_{2}$$

simplifies to

$$n^{2} - 5n - 300 = 0$$

[3 marks]

7 (b) (ii) Hence, solve the equation

$$2 \times {}^{n}C_{4} = 51 \times {}^{n}C_{2}$$

[2 marks]